

Weighted Tree Similarity Based Optimization for Agent Matching in Multi Agent Markets

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Abstract – The article analyzes the nature of multi-agent markets and suggests a new framework based on hierarchical matching and decision architecture. Furthermore, the framework adopts weighted tree similarity algorithms to obtain a more detailed perspective of the agents' interaction and preferences. The article describes the theoretical background of fuzzy similarity measures based on trees and introduces a new weighted tree similarity algorithm that enables assessment of the agent preferences and characteristics by employing fuzzy representations and hierarchy. Therefore, it fosters better and specific matching decisions that can be made. In addition, the article examines whether and how the Stackelberg equilibrium, a form of game theory, can help to enhance the efficiency of the interactions and decisions of the agents. The main conclusions in this article include the creation of the new weighted tree similarity algorithm for multi-agent markets, the creation of the hierarchical matching and decision system, and the verification that the proposed system significantly improves the efficiency of interactions between agents in the market. The effectiveness of these conclusions is supported by case and empirical evidence, which further emphasizes the applicability and the possibility of the projected framework in real-world market scenarios. This is beneficial in providing significant information concerning the improvement of the market forces and decision making.

Keywords – Multi-Agent Markets, Weighted Tree Similarity, Multi-Agent Systems, Hierarchical Matching, Fuzzy Representations, Game Theory, Market Efficiency.

I. INTRODUCTION

The rapid expansion of the Internet in contemporary society has led to a significant growth in electronic commerce. Currently, it is possible to implement multi-agent structures where computer programs function as intermediaries, vendors, and purchasers. These agents have the ability to engage in negotiations and execute bilateral agreements on our behalf. Typically, the relationships between representatives in a multi-agent marketplace are intricate. Every buyer and seller agency wants to engage in transactions with managers that exploit their individual payouts. In order to accomplish this, three essential elements are required: (1) A depiction of the products or services that includes the specific preferences of the agents for each item; (2) A metric that determines which agents are capable of fulfilling the desired products or services of other agents in the market; and (3) A system for facilitating connections among sellers and buyers in order to increase their financial gains.

The agents could encounter multi-feature goods that exhibit interrelations between their qualities. Various research studies have examined attribute interdependencies from different perspectives. Durbach and Stewart [1] have employed nonlinear utility functions to model agents' preferences, and hierarchical models, such as the tree representation model, have been utilized in multiple studies to analyze multi-attribute products. The authors examined the price changes in a basic model of the shopbot economy put out by Greenwald and Kephart. This model assumes a market with a large quantity of

sellers (S) and a relatively smaller amount of customers ($B \gg S$). The sellers compete to offer a single, indivisible item, such as a specific book, to the buyers. Every buyer possesses a valuation cap, which is the highest price they are eager to pay for the item. Buyers can be classified into two categories based on their search method for a seller: (i) A buyer who is actively seeking bargains uses a shopbot to choose the seller that offers the lowest price. The buyer will only make a purchase if the price offered by the seller is below a certain price threshold called p_{cap} . (ii) A buyer who selects sellers randomly will make a purchase if the price offered by the seller is below a specific price limit called p_{cap} .

In this model, the objective of a seller is to optimise its immediate profit by determining a price p_t for a single unit of the commodity, which incurs a production cost of p_{co} . In the current economic climate, sellers are motivated to reduce their prices in order to gain the largest possible portion of the market. If a seller is not providing the most competitive pricing, then its optimal approach is to offer the highest price that the market can tolerate. Considers the issue of price setting as a single occurrence game and demonstrates that pricebots employing either a myoptimal or a no-regret pricing strategy result in recurring price conflicts. The objectives of the agents and the dynamics of market demand and supply are crucial considerations for making decisions in multi-agent marketplaces. Instantaneous decision-making results in struggle between the agents, where the decision made by one agent immediately impacts the aims of other agents and their subsequent choices. Hence, the matter of coordination in such a setting is an essential undertaking. Game theory can be employed to effectively handle conflicts arising from decision-making. Garcia et al. [2] introduced a bidding technique based on Nash equilibrium for bilateral contracts in power markets. A matching procedure is conducted between the generators and loads in order to minimize the overall generating costs in a power model. Nevertheless, the stability point obtained is not a genuine Nash solution as it fails to meet certain Nash inequalities. The initial identification of this issue was made by Ratliff, Burden, and Sastry [3]. In the approach described by Heilman et al. [4], a subjective rank ordering of each decision maker's choices relative to those of other decision makers takes the place of a reward function.

Furthermore, the utilization of the ordinal game idea was employed to acquire generalized and optimal Nash stability points. In their study, Wang, Wu, and Liu [5] provided a concise overview of the findings in the ordinal approach and discussed several applications in the fields of business and management. Another use of ordinal games for seller-buyer agency matching was documented by Kebriaei Majd, and Rahimi-Kian [6]. Decentralized pairing can be employed when there is multiple buyers and a single seller, or multiple sellers and a single buyer. In such scenarios, a single seller or buyer assumes the role of a leader, aiming to find the most comparable counterpart and achieve a state of balance in the game. Nevertheless, in a multi-buyer/multi-seller game, it is advisable to appoint a broker or representative to oversee the matchmaking procedure in order to prevent decision conflicts among the agents. Without proper management, the matchmaking may happen randomly, leading to suboptimal outcomes. In the centralized system, the most comparable seller/buyer is paired in a leader-follower game, notwithstanding the agents' restriction to following the representative agent.

This article discusses the urgent need for improved decision-making frameworks in multi-agent marketplaces. The objective is to enhance agent interactions and maximize market efficiency by using hierarchical matching, weighted tree similarity algorithms, and game theory. Conventional methods often lack the subtlety necessary for precise matching and decision-making, resulting in less-than-ideal results. The subsequent sections of the research have been arranged as follows: Section II presents the conceptual analysis of weighted tree similarity, fuzzy tree measure, simple center of gravity method, radius of gyration-based similarity measure, and geometrical mean averaging operator similarity measure. Section III provides a description of new decisions, and agent-matching architecture. This section describes a novel weighted tree similarity algorithm, and decision-making using game theory. Section IV provides a description of the case study to better understand optimizing agent matching in multi-agent markets using weighted tree similarity. Section V concludes the article by providing a summary of the discussions in the paper.

II. CONCEPTUAL ANALYSIS

Weighted Tree Similarity

In this segment, we offer a concise outline of notations and terminology, drawing from established literature. We then proceed to discuss the weighted tree similarity technique that has been previously proposed [7]. The initial algorithm is shown utilizing Relfun, while we provide a more comprehensive specification.

Notation and Definitions

Consider T and T^1 as the bases of two trees. Assume that the root knot of T includes m , $l_1, l_2, \dots, l_m$, and that the sub-tree connected to arc l_j is showed by the sub-root node T_i , where i ranges from 1 to m . For each j from 1 to n , the sub-tree connected to arc T_j^i is denoted as T_j^i , and the root node of T^i is represented by n arcs denoted as $l_1^i, l_2^i, \dots, l_n^i$. The arc weight l_i is represented by $w(l_i)$ while the arc mass l_j^i is represented by $w(l_j^i)$. Let Node-tag(T) be the representation of the node label. The two trees' node labels can be compared recurrently to determine how similar they are to one another.

Definition 1. The tree is normalized and has arcs labelled with arcs weights [8]. 5 tuples (V, E, L_V, L_E, L_W) make up a regularized arc-labeled, arc-weighted tree. In this type of tree, each tuple represents a different set of information: the vertices number (V), the edges number (E), the labels (L_V), the weights (L_W), which can be anything from 0 to 1. Making use of a fan-out-mass-regularized technique with $n \rightarrow 1$, where n is an integer from 1. As you map the aspects in E in L_W , make sure that the total of each fan-out's weights equals 1.0.

Comparison System for Weighted Tree

The method performs a recurrent top-down contrast of two provided regularized arc-weighted, arc-characterized trees (as defined in Definition 1). This computation yields a comparison value ranging from 0 to 1. If the two trees and their corresponding ideas (i.e., complete edifices) are identical, then the comparison is 1. However, if the root notions of the two trees vary, the comparison is 0. The similarity value for all other situations falls within the range of 0 to 1. Initially, two provided trees' root node labels are compared using the weighted tree similarity method (refer to method 1). If the labels are not same, the similarity is returned as zero.

Algorithm 1: wT -similarity (T, T')

Input: Two trees T and T'

Output: A similarity score between 0 and 1

```

1. Initialize match = 0,  $\lambda value = 0$ ,  $0 \leq \varepsilon \leq 1$ 
2. if Node-label ( $T$ ) = Node-label ( $T'$ ) then
3.   if  $T$  and  $T'$  are leaf nodes then
4.     Return 1
5.   else
6.     if  $T$  and  $T'$  are non-leaf nodes then
7.       for each pair of identical arc labels  $l_i \in T$  and  $l_j \in T'$  do
8.          $match = match + \frac{w(l_i) + w(l_j)}{2} * wT - similarity(T_i, T_j)$ 
9.       end for
10.      for each arc  $l_i$  present in  $T$  and does not exist in  $T'$  do
11.         $\lambda value = \lambda value + \frac{w(l_i)}{2} * \lambda(1.0, T_i)$ 
12.      end for
13.      for each arc  $l_j$  present in  $T'$  and does not exist in  $T$  do
14.         $\lambda value = \lambda value + \frac{w(l_j)}{2} * \lambda(1.0, T_j)$ 
15.      end for
16.      Return ( $\varepsilon + (1 - \varepsilon) * (match + \lambda value)$ )
17.    else if  $T$  is a non-leaf and  $T'$  is a leaf then
18.      for each arc  $l_i \in T$  do
19.         $\lambda value = \lambda value + \frac{w(l_i)}{2} * \lambda(1.0, T_i)$ 
20.      end for
21.      Return ( $\varepsilon + (1 - \varepsilon) * (\lambda value)$ )
22.    else if  $T$  is a leaf and  $T'$  is a non-leaf then
23.      for each arc  $l_j \in T'$  do
24.         $\lambda value = \lambda value + \frac{w(l_j)}{2} * \lambda(1.0, T'_j)$ 
25.      end for
26.      Return ( $\varepsilon + (1 - \varepsilon) * (\lambda value)$ )
27.    end if
28.  end if
29. else
30.   Return 0
31. end

```

It is important to observe that the similarity of the weighted tree algorithm calculates two similarity components. If the labels of the two nodes are the same, then the initial portion contributes a little value to the similarity, denoted by $0 < \varepsilon < 0.5$. The sub-trees of these two nodes are circular compared for similarity, which is then multiplied by $1 - \varepsilon$. This result constitutes the second component, which is then added to the first part. This approach prevents the recursive technique from yielding a similarity score of zero when the root knot tags of two trees are the similar but all child tags are dissimilar. In such cases, it ensures a modest value owing to the presence of undistinguishable node labels. The comparison in the aforementioned technique is calculated using a circular top-down diagonal of all consistent nodes in both trees, as outlined in Algorithm 1.

Simplicity Algorithm for Missing Trees

This segment elucidates the technique's straightforwardness [9]. When one of the trees being compared is incomplete, the simplicity method will calculate the missing tree's contribution. Guindon et al. [10] proposed a proposal in which the simplicity technique relies on the topology of the missing tree. The missing tree T and a simplicity factor (r), which establishes the absent tree's initial contribution to the comparison calculation, are the two parameters needed for the similarity

computation. This method returns a value inside the interval from 0 to r . The tree's simplicity value is r due to the absence of a single node. The depth reduction factor is used to trees that grow vertically to decrease their simplicity value, which is fixed at 0.5. If the tree extends in a horizontal manner, calculate the subjective average of the simplicity functions of all the child sub-trees. If a tree continues to grow both horizontally and vertically indefinitely, its simplicity value will ultimately converge to zero. The approach for the simplicity functionality is described in [11]. Equation (1) to (18) provides a recursive simplicity function.

$$\lambda(r, T) = \begin{cases} \frac{1}{m} \sum_{j=1}^m w(l_j) & \text{if } T \text{ is a leaf} \\ \lambda(rd, T_j) & \text{otherwise} \end{cases} \quad (1)$$

where $\lambda(r, T)$ refers to the simplicity function, r is the aspect of simplicity, d is intensity deprivation aspect = 0.5, m arcs for a tree T stated as l_j , for $j = 1$ to m . T_j refer to the subtree linked to arc l_j , $w(l_i)$ is the arc weight l_i , for $j = 1$ to m .

Fuzzy Similarity Measure

For any two-trapezoidal fuzzy numbers $A = (a_1, \dots, a_4)$ and $B = (b_1, \dots, b_4)$, (2), (3), and (4) may be considered.

$$S(A, B) = 1 - \frac{\sum_{i=1}^4 |a_i - b_i|}{4} \quad (2)$$

$$S(A, B) = \frac{1}{1+d(A, B)} \text{ Where } d(A, B) = |P(A) - P(B)| \quad (3)$$

$$P(A) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6} \quad P(B) = \frac{b_1 + 2b_2 + 2b_3 + b_4}{6} \quad (4)$$

Simple Center of Gravity Method

The SCGM (simple center of gravity method) is founded on the principle of medium curve [12]. The SCGM approach combines COG distance of GFN's and the principles of geometric distance, in consideration to equations (5)-(15). If the GFN's are $A = (a_1, \dots, a_4; w_A)$ and $B = (b_1, \dots, b_4; w_B)$ $0 \leq a_1, \dots, a_4 \leq 1$ and $0 \leq b_1, \dots, b_4 \leq 1$.

$$\text{COG}(A) = (x_A^*, y_A^*), \quad \text{COG}(B) = (x_B^*, y_B^*) \text{ then} \quad (5)$$

$$S(A, B) = 1 - \frac{\sum_{i=1}^4 |a_i - b_i|}{4} (1 - |x_A^* - x_B^*|, B(S_A, S_B) \frac{\min(y_A^*, y_B^*)}{\max(y_A^*, y_B^*)}) \quad (6)$$

where

$$y_A^* = \begin{cases} \frac{w_A(a_3 - a_2 + 2)}{6} & \text{if } a_1 \neq a_4 \\ \frac{w_A}{2} & \text{if } a_1 = a_4 \end{cases} \quad (7)$$

$$x_A^* = \frac{y_A^*(a_3 + a_2) + (a_4 + a_1)(w_A - y_A^*)}{2w_A} \quad (8)$$

$$B(S_A, S_B) = \begin{cases} 1 & \text{if } S_A + S_B > 0 \\ 0 & \text{if } S_A + S_B = 0 \end{cases} \quad (9)$$

$$S_A = a_4 - a_1 \quad S_B = b_4 - b_1 \quad (10)$$

The Radius of Gyration-Based Similarity Measure

$$S(A, B) = 1 - \frac{\sum_{i=1}^4 |a_i - b_i|}{4} (1 - |r_x^A - r_x^B|)^{B(S_A, S_B) \frac{\min(r_x^A, r_x^B)}{\max(r_x^A, r_x^B)}} \quad (11)$$

Where;

$$r_x^A = \sqrt{\frac{(l_x)_1 + (l_x)_2 + (l_x)_3}{((a_3 - a_2) + (a_4 - a_1)) \frac{w_A}{2}}} \quad r_y^A = \sqrt{\frac{(l_y)_1 + (l_y)_2 + (l_y)_3}{((a_3 - a_2) + (a_4 - a_1)) \frac{w_A}{2}}} \quad (12)$$

$$(l_x)_1 = \frac{(a_2 - a_1)w_A^3}{12} \quad (l_x)_2 = \frac{(a_3 - a_2)w_A^3}{3} \quad (l_x)_3 = \frac{(a_4 - a_3)w_A^3}{12} \quad (13)$$

$$(l_y)_1 = \frac{(a_2-a_1)^3 w_A}{4} + \frac{(a_2-a_1)a_1^2 w_A}{2} + \frac{2(a_2-a_1)^2 a_1 w_A}{3} \quad (14)$$

$$(l_y)_2 = \frac{(a_3-a_2)^3 w_A}{3} + \frac{(a_3-a_2)a_2^2 w_A}{1} + \frac{(a_3-a_2)^2 a_2 w_A}{1} \quad (15)$$

$$(l_y)_3 = \frac{(a_4-a_3)^3 w_A}{12} + \frac{(a_4-a_3)a_3^2 w_A}{2} + \frac{2(a_4-a_3)^2 a_3 w_A}{3} \quad (16)$$

Similarity Measure According to the Geometrical Mean Averaging Operator

$$S(AB) = \left[\sqrt[4]{\prod_{i=1}^4 (2 - |a_i - b_i|) - 1} \right] \times \frac{\min(y_A^*, y_B^*)}{\max(y_A^*, y_B^*)} \quad (17)$$

$$\text{where } y_A^*, y_B^* \text{ are given by} \quad (18)$$

III. A NEW DECISION AND AGENT MATCHING ARCHITECTURE

Our approach incorporates a hierarchical framework of supply or demand characteristics for each agent. The concept is derived on the model proposed by Lampropoulos, Alsaikaf, and Blom [13]. However, based on the comments in Section II, we choose for fuzzy labels instead of crisp depiction of feature values to enable a more accurate assessment of the anticipated qualities. We furthermore provide a novel fuzzy similarity metric to calculate a comprehensive resemblance between every vendor and buyer pair. Unlike traditional actions, which are all symmetrical, the suggested action allows sellers and buyers to have differing perspectives on one other, based on distinct attribute values. The suggested similarity measure among every pair of the membership purposes of the buyer and seller is designed to include the benefits of measures (5)-(8) while mitigating the disadvantages described in Section II. Subsequently, we use game theory to facilitate decision-making for the agents. Similarity values in this game serve as the agents' payoffs. Since the buyer and seller representatives react to offers and bids from their rivals successively, the Stackelberg approach is a stronger matching game strategy than the Nash solution.

Novel Weighted Trees Fuzzy Similarity Algorithm

This segment introduces a method for measuring the comparison among seller and buyer trees in a multiple agent market, using an ordered approach. The intended values in an agent's fuzzy representation of products/services are characterized by membership purposes located at the lower part of the tree. **Fig. 1** illustrates an instance of a tree in an energy sector that consists of three tiers.

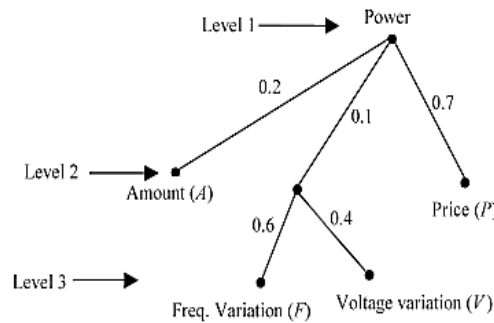


Fig 1. An Illustration of an Agent's Depiction of a Tree.

The sets A, F, V, and P are illustrated using the units of quantity, voltage disparity, price, and frequency fluctuation, to describe their fuzzy properties. The agent may either use a pre-established collection of words or autonomously design the association purposes for these fuzzy sets according to their preferences. For instance, fuzzy set A may accept certain terms like "Medium," "High," or "Low" based on its membership function μ_A . **Fig. 1** shows the seller's features with P levels, while another graph shows the i^{th} knot at the k^{th} stage of the seller's tree as $t_{k,i}^S$ and the buyer's tree as $t_{k,i}^B$. Equations (19) provides a recursive formula to explain the assessment of fuzzy similarity from the viewpoint of the seller for nodes $t_{k,i}^S$ and $t_{k,i}^B$:

$$\text{Sim}(t_{k,i}^S, t_{k,i}^B) = \begin{cases} \frac{\sum_{j=1}^i v_{ij}^S (1 - |\mu_{t_{k,j}^S}(x_j) - \mu_{t_{k,j}^B}(x_j)|)}{\sum_{j=1}^i v_{ij}^S} & \text{if } t_{k,j}^S, t_{k,i}^B \text{ are leaf nodes,} \\ \frac{\sum_{j=1}^{n_i} w_{ij}^S \cdot \text{Sim}(t_{k+1,j}^S, t_{k+1,j}^B)}{\sum_{j=1}^{n_i} w_{ij}^S} & \text{otherwise.} \end{cases} \quad (19)$$

Let k be a variable that ranges from 1 to P . The important weight for the j^{th} element of attribute x , denoted as $v_{ij}^S = \mu_{t_{k,i}^S(x_j)}$, represents the significance of this element. The superscripts “S” and “B” refer to the buyer and seller. The variable x_j represents a fuzzy variable that may take on l different values. The term w_{ij}^S refers to another variable. Within the buyer's and seller's trees, S represents the mass of the division that extends from the i^{th} node at phase k to the j^{th} node at phase $k + 1$. On the other hand, n_i is the sum of sections at level k for node i . Given that the above definition is a sequential comparison, we could also establish the purchaser's fuzzy similarity evaluation for the seller at node $t_{k,i}^S, t_{k,i}^B$ as provided in Equation (20).

$$\text{Sim}(t_{k,j}^B, t_j^S) = \begin{cases} \frac{\sum_{j=1}^i v_{ij}^B (1 - |\mu_{t_{k,i}^B}(x_j)|)}{\sum_{j=1}^i v_{ij}^B} & \text{if } t_{k,i}^S, t_{k,j}^B \text{ are leaf nodes,} \\ \frac{\sum_{j=1}^{n_j} w_{ij}^B \cdot \text{Sim}(t_{k+1,j}^B, t_{k+1,j}^S)}{\sum_{j=1}^{n_j} w_{ij}^B} & \text{otherwise.} \end{cases} \quad (20)$$

Due to the presence of just one node at level 1, we refer to the comparison at that node as generic comparison. The seller's perspective defines the broad resemblance between customer B and seller S as provided in Equation (21).

$$\text{Sim}(S, B) = \text{Sim}(t_{1,1}^S, t_{1,1}^B) \quad (21)$$

Similarly, the overall resemblance in the perspective of the customer is referred considering Equation (22).

$$\text{Sim}(B, S) = \text{Sim}(t_1^B, 1, t_1^S) \quad (22)$$

The weights v_{ij}^S and v_{ij}^B are used in comparison meanings (16) and (17), correspondingly, to ascertain the importance of items with a higher (lower) degree of membership. As stated in [14], nonsymmetric weighted similarity metrics provide more flexibility in design and better alignment with real-world situations.

Decision Making Using Game Theory

Applying Game Theory to Attain a State Of Equilibrium

Tuyls and Parsons [15] provide the results acquired from their built model and evaluate if these results are consistent with the expected behavior of rational entities and players. The units in the created framework consist of distributors of the primary brand, clients, and e-tailers. The model's output was verified for agreement with the five facts and suggestions presented in **Table 1**. In order to execute the given model, it is necessary to adjust its starting parameters. The needed variables are determined using the values shown in **Table 2**. The process of determining the game balance point is explained in detail in **Table 3-5**.

Table 4 displays the optimal response functionality for the e-tailer based on the dealer's choice on fixed and variable expenses, in relation to quantity, price, and assortment size. Dealer profit is determined by analyzing the e-tailer's response to its actions. Ultimately, the solution that yields the most profit for the provider is chosen, as seen in **Table 3**. **Table 4-5** reveal the e-tailer's feedback and related profit by analyzing the distributor's offerings about assortment cost. **Table 4** displays the decision metrics of the e-tailer in relation to quantity, price, and assortment size. On the other hand, **Table 5** presents e-tailer profit for each grouping of variable and fixed costs.

Table 1. Validation of the Framework using 5 Facts

#	Statement
Fact 1	Reducing the cost of purchasing alternative items decreases the usability of the primary label for e-tailers.
Fact 2	By incorporating the profitability of the carrier as a factor in the network, the online retailer is willing to pay a higher price for acquiring the leading brand if it generates profit for the retailer.
Fact 3	If the profit generated by the related main item basket exceeds the anticipated value, the e-tailer will reduce the gross profit on the feature in order to retain customers who purchase the entire basket.
Fact 4	The final buying choice of the e-tailer is influenced by the distribution of two client kinds.
Fact 5	An e-tailer experiences fewer profits when there is a greater sensitivity of demand to pricing.
Recommendation	The model proposes increasing the variety of products in response to a high level of price sensitivity.

Table 2. Amount to Initialize the Necessary Parameters

Parameters	Values
N	6
($u_1, u_2, \dots, u_6$)	[5, 5, 5, 5, 5, 5]
u_0	0.9
h	$0.1 \cdot C$
λ, Q	100.1

Table 3. Profit Function Leader

Supplier-objective function		Fixed costs (c \$)					
		0	10	20	30	40	50
Variable costs (c \$)	1	67.3701	101.2426	113.2815	133.2819	115.0924	125.0924
	1.5	101.123	150.7868	154.2225	174.2225	194.2225	164.8799
	2	151.763	203.2186	215.6904	216.2106	236.2106	256.2106
	2.5	198.8193	258.8193	280	289.9174	278.6723	298.6723
	3	247.8287	307.8287	347.4845	334.8238	321.1372	341.1372

Table 4. Best Reaction to Choices About (Quantity, Selling Price, Assortment Size) Made by Followers (E-Tailers)

Best retort (quantity, selling price, assortment size) decision of the e-tailer							
		Fixed cost					
		0	10	20	30	40	50
Variable cost (Sc)	1	(5, 4.92, 13.77)	(4, 4.49, 23.75)	(2, 4.16, 36.64)	(2, 4.16, 36.64)	(1, 3.59, 75.09)	(1, 3.59, 75.09)
	1.5	(6, 5.15, 11.90)	(4, 4.81, 18.46)	(2, 4.25, 38.07)	(2, 4.25, 38.07)	(2, 4.25, 38.07)	(1, 3.69, 76.59)
	2	(6, 5.24, 12.65)	(5, 5.09, 15.32)	(3, 4.67, 25.95)	(2, 4.35, 39.05)	(2, 4.35, 39.05)	(2, 4.35, 39.05)
	2.5	(6, 5.33, 13.25)	(6, 5.33, 13.25)	(4, 5.00, 20.00)	(3, 4.77, 26.66)	(2, 4.45, 39.73)	(2, 4.45, 39.73)
	3	(6, 5.42, 13.77)	(6, 5.42, 13.77)	(5, 5.28, 16.50)	(3, 4.88, 27.20)	(2, 4.57, 40.19)	(2, 4.57, 40.19)

Table 5. Profit Functions of the E-Tailer; E-Tailer Objective

Variable cost (Sc)	Fixed cost (Sc)					
	0	10	20	30	40	50
1	429.83	394.02	371.55	351.55	339.36	329.36
1.5	436.57	386.59	356.56	336.56	316.56	306.45
2	432.31	374.96	339.45	315.44	295.44	275.44
2.5	420.52	360.52	316.86	279.39	252.72	232.72
3	371.83	311.83	254.38	218.41	194.50	174.50

An advantageous solution for games is when one player possesses the capacity to declare their option prior to the other player. Under this supposition, the leader can optimize its plan by following the potential responses of the follower. Subsequently, the follower identifies the most advantageous response to the leader's action. The suggested comparison measure is utilized to compute the similarity value between each vendor and buyer pair. The Stackelberg game in **Algorithm 2** is employed to address this difficulty, as each agent aims to optimize its payoffs while also seeking its most closely aligned counterpart in terms of objectives and desires.

Algorithm 1: Stackelberg (C, c)

Input:

set of feasible fixedCosts: feasible_fixedCost,
set of feasible variableCosts: feasible_variableCost

Output:

Optimized fixed and variable costs: C, c

1. Initialize i, j to 0
2. for each C in feasible_fixedCost do
3. $j = j + 1$
4. for each c in feasible_variableCost do

```

5. [S, P, X] = followersub_problem
6.     i = i + 1
7.     Supplier_profit [i, j] = leadersub problem (S, X)
8.   end for
9. end for
10. t = Argmax (Supplier_profit[:, j])
11. h = Argmax (Supplier_profit[:, j])
12. [C, c] = feasible_fixedCost_t, feasible_variableCost_h
13. Return Optimized fixed_variable costs: C, c

```

Consider $(\text{Sim}(B_i, S_j))$ $\text{similarity}(B_i, S_j)$ as the overall similarity between the j^{th} sellers and the i^{th} buyers from the buyer's perspective, and $\text{Sim}(S_j, B_i)$ as the overall similarity among the j^{th} sellers and the i^{th} buyers from the seller's perspective. The game matrix representing the matchmaking problem is illustrated in **Table 6**. Let B represent the demonstrative who represents the buyers, and S represent the agent who represents the sellers. The decision spaces for S and B are outlined using Equation (20).

$$\hat{W} = \{B_1, B_2, \dots, B_n\}, \hat{W} = \{S_1, S_2, \dots, S_m\} \quad (23)$$

Put simply, each decision maker has the ability to select a seller or buyer to participate in market negotiations, depending on the potential benefits they might gain from the game matrix. Each decision pair $\{\{B_i \in \hat{W}^B, S_j \in \hat{W}^S\}$ yields the subsequent payouts for B (Buyer) and S (Seller) (see **Table 6**).

Table 6. Payoff Game Matrix

Buyer	Seller			
	S_1	S_2	...	S_3
B_1	Similarity (S_1, B_1) Similarity (B_1, S_1)	Similarity (S_2, B_1) Similarity (B_1, S_2)	...	Similarity (S_m, B_1) Similarity (B_1, S_m)
B_2	Similarity (S_1, B_2) Similarity (B_2, S_1)	Similarity (S_2, B_2) Similarity (B_2, S_2)	...	Similarity (S_m, B_2) Similarity (B_2, S_m)
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
B_3	Similarity (S_1, B_n) Similarity (B_n, S_1)	Similarity (S_2, B_n) Similarity (B_n, S_2)	...	Similarity (S_m, B_n) Similarity (B_n, S_m)

$$J^B(B_i, S_j), J^S(B_i, S_j) = \text{Sim}(S_j, B_i) \quad (24)$$

The reaction functionality of agent S, as defined by Riehl, Ramazi, and Cao [16], describes how agent S responds to agent B's action while choosing B_i .

$$R^S(B_i) = \{\xi \in r^s \mid \text{Sim}(\xi, B_i) \geq \text{Sim}(S_j, B_i) \forall S_j \in r^s\} \quad (25)$$

The expression $R^S(B_i)$ represents the set of reactions of agent S in retort to the act B_i performed by agent B. The variable ξ belongs to agent S's strategy set. Hence, the most favorable choice B_i^* for choice maker B is the resolution of Equation (24 to 26).

$$\text{Sim}(B_i^*, R^S(B_i^*)) = \max \text{Sim}(B_i, R^S(B_i)) = J^{B^*} \quad (26)$$

The ideal choice for agent S, represented as S_j^* , is equivalent to $R^S(B_i^*)$. Therefore, the choice pair $\{B_i^* \in \hat{W}^B, S_j^* \in \hat{W}^S\}$ represents the Stackelberg model of the game, whereas $\{\text{Sim}(B_i^*, S_j^*), \text{Sim}(S_j^*, B_i^*)\}$ denotes the associated Stackelberg equilibrium payoffs. After identifying the equilibrium point, the matching pair is eliminated from the choice space. The equal process is then carried out for the residual factors using a lower dimension's game matrix. The aforementioned technique is continued iteratively until all the associates of either the vendors or buyers' group are successfully paired.

IV. CASE STUDY

Fig. 2 displays the attribute trees for Buyer 1 and Seller 1. This study evaluates four major factors as indicators of power quality. These concerns include price, quantity, reliability, and quality. **Fig. 2** displays the weights associated with the tree branches. To further analyze each of the given criteria, we may assign a relative value to its individual components. In order

to clarify, the buyer's dependability factor consists of three sub-factors: the Average Annual Outage Duration (AAOD), the Average Outage Length (AOD), and the Average Failure Rate (AFR). The sub-factors are assigned weights of 0.25, 0.25, and 0.5.

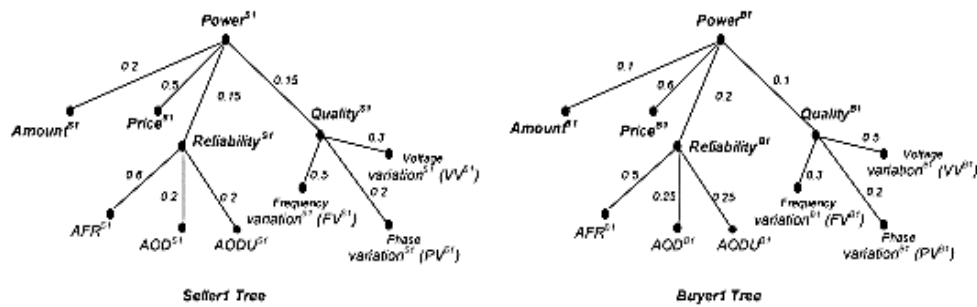


Fig 2. Attribute Trees for Sellers and Buyers in the Power Market.

Table 7. Contrast among Buyer 1 and Seller 1 from Seller's Perspective at Level 3

Sim (AFR ^{S1} , AFR ^{B1}) 0.512	Sim (AODU ^{S1} , AODU ^{B1}) 0.453	Sim (AOD ^{S1} , AOD ^{B1}) 0.633
Sim (FV ^{S1} , FV ^{B1}) 0.253	Sim (VV ^{S1} , VV ^{B1}) 0.421	Sim (PV ^{S1} , PV ^{B1}) 0.491

Table 8. Comparisons at Level 2

Similarity (Amount ^{S1} , AFR ^{B1})	Similarity (Reliability ^{S1} , Reliability ^{B1})	Similarity (Price ^{S1} , Price ^{B1})	Similarity (Quality ^{S1} , Quality ^{B1})
0.763	0.471	0.833	0.69

Subsequently, we will proceed to execute the procedures delineated in Section III in order to compute the weighted tree similarity between the providers and buyers. Fig. 2 illustrates that the buyer and seller trees contain distinct sets of leaves, where fuzzy sets with proportional membership purposes are established. For instance, the seller's membership functions for price are represented as μP^{S1} , whereas the buyer's membership functions for price are marked as μP^{B1} . The similarities between the vendor and consumer associated with each level are determined by using equations (8) and (9). The comparison ethics among Buyer 1 and Seller 1 at Level 2 and Level 3, are shown in Table 7 and 8, from the seller's viewpoint. Finally, from the viewpoint of the seller, the level 1 similarity between the powers of S^1 and B^1 is calculated as $\text{Sim}(Power^{S1}, Power^{B1}) = 0.616$. There is a bimatrix game that arises from implementing this strategy to every conceivable combination of buyers and sellers from all possible viewpoints. Matchmaking may now be conducted using the decision-making methodology outlined in Section III. Equation (24) states that decision spaces exist for S and B.

$$\hat{W} = \{B_1, B_2, \dots, B_{10}\}, \hat{W} = \{S_1, S_2, \dots, S_6\} \quad (27)$$

Table 9. Response of the Supporter to the Actions of the Leader

Buyer _i	Buyer ₁	Buyer ₂	Buyer ₃	Buyer ₄	Buyer ₅	Buyer ₆	Buyer ₇	Buyer ₈	Buyer ₉	Buyer ₁₀
R ^S (Buyer _i)	Seller ₄	Seller ₃	Seller ₆	Seller ₃	Seller ₅	Seller ₃	Seller ₁	Seller ₃	Seller ₆	Seller ₂

Table 10. Match-Making Based on Comparisons as Payoffs

	Phase 1	Phase 2	Phase 3	Phase 4	Phase 5	Phase 6
Decision pairs	(Buyer ₄ , Seller ₃)	(Buyer ₂ , Seller ₂)	(Buyer ₁₀ , Seller ₆)	(Buyer ₃ , Seller ₁)	(Buyer ₇ , Seller ₅)	(Buyer ₁ , Seller ₄)
Payoff	(0.75, 0.63)	(0.76, 0.69)	(0.76, 0.74)	(0.96, 0.65)	(0.96, 0.55)	(0.68, 0.94)

The reaction function of S may be derived from equation (24) and is shown in Table 9. By using equation (24), the most favorable choice for B is B_4 . Similarly, based on $R^S(B_i)$, the most advantageous option for S is S_3 . The Stackelberg equilibrium is determined to be the choice pair $\{B_4, S_3\}$, with a matching payout of $\{0.754, 0.632\}$. To eliminate biases from the other sellers and buyers, B_4 and S_3 are removed from the choice set, and the game is repeated. When this technique is used for the sixth time, the matching process is completed and the results are given in Table 10.

V. CONCLUSION

This research promotes an understanding of multi-agent markets and proposes a novel approach using the hierarchical matching technique, the weighted tree similarity algorithms, and game theory. By conducting a critical review of the theoretical background and introduction of a new weighted tree similarity approach, this work suggests that decision-making in complex markets may be enhanced. The proposed framework includes fuzzy representations and hierarchical structures so that agent preferences and features can be assessed better hence leading to improved matching results. Furthermore, the application of game theory, especially the Stackelberg equilibrium, is a better approach to improving the relations between the agents and the efficiency of the market. The empirical validation and case studies provide evidential support for the practical applicability and usefulness of the proposed framework in a real market environment, which provides evidence of the potential to improve on the current market conditions and decision-making. Lastly, the paper provides key recommendations to overcome major issues in multi-agent marketplaces and presents a realistic approach to improve market effectiveness and best results for buyers and sellers.

CRedit Author Statement

The authors confirm contribution to the paper as follows:

Conceptualization: Anna Recchi and Minu Balakrishnan; **Methodology:** Anna Recchi; **Writing-Original Draft Preparation:** Anna Recchi and Minu Balakrishnan; **Investigation:** Anna Recchi and Minu Balakrishnan; **Supervision:** Minu Balakrishnan; **Validation:** Anna Recchi and Minu Balakrishnan; **Writing- Reviewing and Editing:** Anna Recchi and Minu Balakrishnan; All authors reviewed the results and approved the final version of the manuscript.

Data Availability

The datasets generated during the current study are available from the corresponding author upon reasonable request.

Conflicts of Interests

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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Competing Interests

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